

Snapshot problems for the wave equation and for the Euler-Poisson-Darboux equation

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December 20, 2024

Abstract.

In this talk, we deal with snapshot problems for the wave equation and for the Euler-Poisson-Darboux equation. For simplicity, let us consider the wave equation $\partial_t^2 u - \Delta u = 0$ on \mathbb{R}^n with the condition $u|_{t=t_1} = f_1, \dots, u|_{t=t_m} = f_m$. It is natural to ask when the above equation has a unique solution. We call the above problem the snapshot problem for the wave equation, and the set of m functions $\{f_1, \dots, f_m\}$ the snapshot data.

Roughly speaking, one of our main results is as follows.

Theorem. We assume that $m = 3$ and $(t_3 - t_1)/(t_2 - t_1)$ is irrational and not a Liouville number. In addition, we assume a certain compatibility condition on the snapshot data $\{f_1, f_2, f_3\}$. Then the snapshot problem for the wave equation has a unique solution.

We also consider a similar snapshot problem for the Euler-Poisson-Darboux equation. This is a joint work with Jens Christensen, Fulton Gonzalez, and Jue Wang.