Homeomorphism groups of noncompact 2-manifolds

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Topological properties of homeomorphism groups of compact 2-manifolds and their subgroups have been studied by various authors. In this exposition we report some extensions of these results to the case of non-compact 2-manifolds.

Suppose $M$ is a non-compact connected 2-manifold with $\partial M = \emptyset$. Let $\mathcal{H}(M)$ denote the group of homeomorphisms of $M$ with the compact-open topology. For a subgroup $\mathcal{G}$ of $\mathcal{H}(M)$, let $\mathcal{G}_0$ denote the connected component of $id_M$ in $\mathcal{G}$. The scripts $c$, PL and QC denote “compact support”, “piecewise linear”, “quasiconformal” respectively. Our results are summarized as follows.

**Theorem (1)** ([2], [3]) (i) $\mathcal{H}(M)_0$ is an $\ell_2$-manifold.

(ii) Homotopy Type:

(a) $\mathcal{H}(M)_0 \simeq S^1$ if $M = \mathbb{R}^2$, $S^1 \times \mathbb{R}$ or Open Möbius Band

(b) $\mathcal{H}(M)_0 \simeq *$ in all other cases.

(2) ([4]) $\mathcal{H}^c(M)_0$ is HD (homotopy dense) in $\mathcal{H}(M)_0$.

(3) ([4]) If $M$ is a non-compact connected PL 2-manifold with $\partial M = \emptyset$, then

(i) $\mathcal{H}^{PL,c}(M)_0$ is an $\ell^f_2$-manifold,

(ii) $\mathcal{H}^{PL,c}(M)_0$ is HD in $\mathcal{H}(M)_0$.

(4) ([1]) If $M$ is a connected Riemann surface, then

(i) $\mathcal{H}^{QC}(M)_0$ is a $\Sigma$-manifold,

(ii) $\mathcal{H}^{QC,c}(M)_0$ is HD in $\mathcal{H}(M)_0$.

In [5] we have classified the homotopy type and the topological type of the connected components of the space of embeddings of a compact connected polyhedron into $M$. Recently we have obtained some results on groups of measure-preserving homeomorphisms of noncompact 2-manifolds (math.GT/0507328, 0512231).


