RIMS workshop
Spectral and Scattering Theory and Related Topics

A B S T R A C T

Organizer: Tomoyuki Kakehi (Okayama University)
Coorganizer: Keiichi Kato (Tokyo University of Science)

Day: January 20(Wed.)-22(Fri.), 2016

Venue: RIMS Room 420 (4th floor).

January 20(Wed.) 13:30-14:20 **Kazunori Ando** (Ehime University)

Title: Anomalous localized resonance on smooth domains using spectral properties of the Neumann-Poincaré operators

Abstract

Anomalous localized resonance (ALR) is a phenomenon caused by the interaction between the small inclusion Ω of negative index material and the matrix. We show that anomalous localized resonance occurs not only on core-shell structure but on a simply connected domain Ω; more precisely, ALR occurs on the ellipse in two dimensions, but it does not occur on the ball in the three dimensions. We use the quasi-static approximation and the spectral properties of the Neumann-Poincaré (NP) operator which is a boundary integral operator on $\partial\Omega$. It is known that the NP operator is compact if $\partial\Omega$ is sufficiently smooth. We find that the accumulation speed of the eigenvalues of the NP operator at zero relates to ALR. This is a joint work with Hyeonbae Kang from Inha University.

January 20(Wed.) 14:30-15:20 **Satoshi Ishiwata** (Yamagata University)

Title: Asymptotic expansion of the transition probability for non-symmetric random walks on crystal lattices

Abstract

A locally finite, oriented infinite graph $X = (V, E)$ is called *crystal lattice* if there exists an abelian group $\Gamma \simeq \mathbb{Z}^d$ acting on X freely and the quotient graph $X_0 = \Gamma\backslash X$ is a finite graph. Kotani, Shirai and Sunada ([3], [2], [6]) applied *discrete geometric analysis* to study the long time asymptotics of symmetric random walks on *crystal lattices*. In this talk, we will discuss the asymptotic expansion of the transition probability of non-symmetric random walks on crystal lattices. This is a generalization of Kotani and Sunada [2] and Sunada [5]. This talk is based on a joint work [1] with Hiroshi Kawabi (Okayama University) and Motoko Kotani (Tohoku University).

References

[1] S. Ishiwata, H. Kawabi and M. Kotani: *Long time asymptotics of non-symmetric random walks on crystal lattices*, preprint, http://arxiv.org/abs/1510.05102

[2] M. Kotani and T. Sunada: *Albanese maps and off diagonal long time asymptotics for the heat kernel*, Comm. Math. Phys. **209** (2000), pp. 633–670.

[3] M. Kotani, T. Shirai and T. Sunada: *Asymptotic behavior of the transition probability of a random walk on an infinite graph*, J. Funct. Anal. **159** (1998), pp. 664–689.

[4] T. Sunada: *Discrete geometric analysis*, Analysis on graphs and its applications, Proc. Sympos. Pure Math., **77**, Amer. Math. Soc., Providence, RI, 2008, pp. 51–83.

[5] T. Sunada: Discrete Geometric Analysis, Lecture slide at Institut Henri Poincaré (2002), Humbolt University (2006), Isaac Newton Institute for Mathematical Sciences (2007).

[6] T. Sunada: Topological Crystallography with a View Towards Discrete Geometric Analysis, Surveys and Tutorials in the Applied Mathematical Sciences **6**, Springer Japan, 2013.

January 20(Wed.) 15:40-16:30 **Yoshihisa Miyanishi** (Osaka University)

Title: Remarks on the difference spectrum of semiclassical Schrödinger Operators

Abstract

Eigenvalues of semiclassical Schrődinger operators are discussed. For a Hamiltonian function $H(x,p) : T^*\mathbf{R}^n \to \mathbf{R} \cup \{\pm\infty\}$, the semiclassical Schrődinger operator $Op_\hbar^W(H(x,p))$ is defined by the $\hbar$-Weyl quantization of $H(x,p)$. Under suitable assumptions on H, the spectrum of the semiclassical Schrődinger operator consists of discrete eigenvalues $E_j(h)$ near E. Our main purpose is to study the difference spectrum:

$$\Lambda_\hbar(E,\varepsilon) \equiv \left\{ \frac{E_j(h) - E_k(h)}{h} \;\middle|\; |E_j(h) - E| < \varepsilon, |E_k(h) - E| < \varepsilon \right\}.$$

We establish some relationships between $\Lambda_\hbar$ and the classical mechanics, including Hamiltonians with singular potentials.

January 21(Thu.) 10:00-10:50 **Erik Skibsted** (Aarhus University)

Title: Decay of eigenfunctions of elliptic PDE's

Abstract

Consider a real elliptic polynomial Q on $\mathbb{R}^d$ and the operator $H = Q(-i\nabla) + V$ on $L^2 = L^2(\mathbb{R}^d)$ for a suitable class of real decaying potentials $V = V(x)$. For any eigenfunction $\phi \in L^2$, $(H - E)\phi = 0$, the *global decay rate* is defined as

$$\sigma_{\mathrm{g}} = \sup\{\sigma \geq 0 | \; e^{\sigma|x|}\phi \in L^2\}.$$

If E is not a critical value of Q (and V has sufficient decay) then $0 < \sigma_{\mathrm{g}} < \infty$, and we prove the existence of a pair $(\omega, \xi) \in S^{d-1} \times \mathbb{R}^d$ satisfying the following system of algebraic equations with $\sigma = \sigma_{\mathrm{g}}$:

$$Q(\xi + i\sigma\omega) = E,$$
$$\nabla_\xi Q(\xi + i\sigma\omega) = \mu\omega; \;\mu = \omega \cdot \nabla_\xi Q(\xi + i\sigma\omega).$$

We present various results on a concept of *local decay rate* of a given eigenfunction which may be viewed to be closer to the finer concept of asymptotics at infinity. We show that for rotationally invariant polynomials the local decay rate is directionally independent and given by the global one.

January 21(Thu.) 11:00-11:50 **Dean Baskin** (Texas A & M University)

Title: Asymptotics of scalar waves on asymptotically Minkowski spaces

Abstract

In this talk I will describe an asymptotic expansion for solutions of the wave equation on long-range asymptotically Minkowski spacetimes. The exponents seen in the expansion are related to the resonances of an asymptotically hyperbolic problem at timeline infinity. If time permits, I will describe the proof, which also simplifies the short-range setting. This is joint work with Andras Vasy and Jared Wunsch.

January 21(Thu.) 13:30-14:20 **Matti Lassas** (University of Helsinki)

Title: Scattering in complex geometrical optics
and the inverse problem for the conductivity equation

Abstract

We study Calderon's inverse problem in the two-dimensional case, that is, the question whether the properties of the conductivity function inside a domain can be determined from the voltage and current measurements made on the boundary. We determine the locations of the jumps of the conductivity function. To do this, we introduce a new method based on the propagation of singularities and the scattering of the singularities from the discontinuities of the coefficient functions. While the conductivity equation satisfied by an electrostatic field is an elliptic equation and does not propagate singularities, the associated equations which are used to construct so-called complex geometrical optics (CGO) solutions are of complex principal type. Standard hyperbolic wave equations satisfied by, e.g., acoustic waves, are examples of real principal type equations, which efficiently propagate singularities along one-dimensional characteristics, that is, along rays. Complex principal type equations propagate singularities along two-dimensional bicharacteristic leaves. In the talk we consider the propagation and scattering of these singularities. This is joint work with A. Greenleaf, M. Santacesaria, S. Siltanen and G. Uhlmann.

January 21(Thu.) 14:30-15:20 **Tapio Helin** (University of Helsinki)

Title: Inverse acoustic scattering from random potential

Abstract

We consider an inverse acoustic scattering problem with a random potential. We assume that our far-field data at multiple angles and all frequencies are generated by a single realization of the potential. From the frequency-correlated data our aim is to demonstrate that one can recover statistical properties of the potential. More precisely, the potential is assumed to be Gaussian with a covariance operator that can be modelled by a classical pseudodifferential operator. Our main result is to show that the principal symbol of this covariance operator can be determined uniquely. What is important, our method does not require any approximation and we can analyse also the multiple scattering. This is joint work with Matti Lassas and Pedro Caro.

January 21(Thu.) 15:40-16:30 **Teemu Saksala** (University of Helsinki)

Title: Determination of a Riemannian manifold from the distance functions

Abstract

We consider the problem on an n dimensional manifold N with a Riemannian metric g that corresponds to the travel time of a wave between two points. The Riemannian distance of points $x, y \in N$ is denoted by $d(x, y)$. For simplicity we assume that the manifold N is compact and has no boundary. Instead of considering measurements on boundary, we assume that the manifold contains an unknown open part $M \subset N$ and the metric is known outside this set. When a spontaneously point produces a wave at some unknown point $x \in M$ at some unknown time $t \in \mathbb{R}$, the produced wave is observed at the point $z \in N \setminus M$ at time $T_{x,t}(z) = d(z, x) + t$. These observation times at two points $z_1, z_2 \in N \setminus M$ determine the *distance difference function*

$$D_x(z_1, z_2) = T_{x,t}(z_1) - T_{x,t}(z_2) = d(z_1, x) - d(z_2, x).$$

Physically, this function corresponds to the difference of times at z_1 and z_2 of the waves produced by the point source at (x, t). An assumption there are a large number point sources and that we do measurements over a long time can be modeled by the assumption that we are given the family of functions

$$\{D_x \ ; \ x \in M\} \subset C((N \setminus M) \times (N \setminus M)), \tag{1}$$

We will formulate and prove an uniqueness result related to data (1).

January 21(Thu.) 16:40-17:30 **Serge Richard** (Nagoya University)

Title: Continuity of the spectra for families of magnetic operators on $\mathbb{Z}^d$

Abstract

During this seminar we shall consider families of magnetic self-adjoint operators on $\mathbb{Z}^d$ whose symbols and magnetic fields depend continuously on a parameter. Our aim will be to show that the main spectral properties of these operators also vary continuously with respect to this parameter. The proof is based on an algebraic setting involving twisted crossed product C^*-algebras.

January 22(Fri.) 10:00-10:50 **Trinh Khanh Duy** (Kyushu University)

Title: On global spectral properties of random Jacobi matrices related to Gaussian beta ensembles

Abstract

The matrix models for Gaussian beta ensembles were introduced by Dumitriu and Edelman since 2002. They are symmetric tridiagonal matrices, called Jacobi matrices, whose components are independent and are distributed according to specific distributions. In this talk, I will introduce some new results on the global spectral properties (empirical distributions and spectral measures) of Gaussian beta ensembles as well as of a related infinite Jacobi matrix which is regarded as the limit of Gaussian beta ensembles in the regime that $n \to \infty$ with $\beta n = const$, n being the matrix size.

January 22(Fri.) 11:00-11:50 **Akito Suzuki** (Shinshu University)

Title: Weak limit theorem for an inhomogeneous quantum walk

Abstract

We consider an inhomogeneous quantum walk on $\mathbb{Z}$ given by a unitary evolution U:

$$(U\Psi)(x) = P(x+1)\Psi(x+1) + Q(x-1)\Psi(x-1), \quad x \in \mathbb{Z},$$

where Ψ is a state vector in the Hilbert space $\mathcal{H} = \ell^2(\mathbb{Z};\mathbb{C}^2)$. Let $C(x) = P(x)+Q(x) \in U(2)$ be an inhomogeneous coin operator and S be a shift operator such that $U = SC$. We assume that U has no singular continuous spectrum and there exists a unitary matrix $C_0 = P_0+Q_0 \in U(2)$ such that

$$\|C(x) - C_0\| \leq c_1|x|^{-1-\epsilon}, \quad x \in \mathbb{Z}\setminus\{0\} \tag{2}$$

with positive c_1 and ϵ independent of x. Here $\|M\|$ stands for the operator norm of a matrix $M \in M_2(\mathbb{C})$. Because the condition (2) implies that $U - U_0$ is a trace class operator, the existence and completeness of the wave operator

$$W_+ = \text{s-}\lim_{t\to\infty} U^{-t}U_0^t\Pi_{\text{ac}}(U_0)$$

are proved by using a discrete analogue of the Kato–Rosenblum Theorem. Here $\Pi_{\text{ac}}(U_0)$ is the orthogonal projection onto the subspace of absolute continuity of $U_0 = SC_0$.

Let $\hat{x}(t) = U^{-t}\hat{x}U^t$ be the Heisenberg operator of the position operator $\hat{x}$. Then,

$$\text{s-}\lim_{t\to\infty} \exp\left(i\xi\frac{\hat{x}(t)}{t}\right) = \Pi_{\text{p}}(U) + \exp(i\xi\hat{v}_+)\Pi_{\text{ac}}(U), \tag{3}$$

where $\Pi_{\text{p}}(U)$ is the orthogonal projection onto the direct sum of all eigenspaces of U and $\hat{v}_+ = W_+\hat{v}_0W_+^*$. Here, $\hat{v}_0$ is the velocity operator, obtained first by Grimmett, Janson, and Scudo [*Phys. Rev. E* **69**, 2004], for a homogeneous quantum walk with the evolution U_0. As a consequence of (3), we have the following weak limit theorem. Let X_t be the random variable denoting the position of a quantum walker at time $t \in \mathbb{N}$ with the evolution operator U and the initial state Ψ_0. Then, X_t/t converges in law to a random variable V with a probability distribution

$$\mu_V = \|\Pi_{\text{p}}(U)\Psi_0\|^2\delta_0 + \|E_{\hat{v}_+}(\cdot)\Pi_{\text{ac}}(U)\Psi_0\|^2,$$

where δ_0 is the Dirac measure at zero and $E_{\hat{v}_+}$ the spectral measure of $\hat{v}_+$.

January 22(Fri.) 13:30-14:20 **Masanori Kawamoto** (Kobe University)

Title: Scattering in a periodically pulsed magnetic field

Abstract

We investigate asymptotic behaviors of a charged particle which is in the plane $\mathbb{R}^2$ and is influenced by a time-periodic plused magnetic field $\mathbb{B}(t) = (0, 0, B(t))$, where the plused magnetic field is orthogonal to the plane, and $B(t)$ satisfies

$$B(t) = \begin{cases} B, & t \in \bigcup_{n\in\mathbb{Z}} I_{B,n} \\ 0, & t \in \bigcup_{n\in\mathbb{Z}} I_{0,n} \end{cases}, \quad \begin{cases} I_{B,n} = [nT, nT + T_B) \\ I_{0,n} = [nT + T_B, (n+1)T) \end{cases}$$

where $B > 0$ and $0 < T_B < T$. T is the period of $\mathbb{B}(t)$. By the virtue of a simplicity of the magnetic field, asymptotic behaviors of a free particle can be calculated exactly, for $t \to \infty$. From this we find that the charged particle has three types of asymptotic behaviors for large t: The particle being in compact region for every $t \gg 1$, the particle acting like linear uniform motion, and the particle being accelerated and the asymptotic velocity of the particle increasing *exponentially* about t. These statements are completely determined by B, T_B, T, q and m, where $q \notin \mathbb{R}\backslash\{0\}$ and $m > 0$ are the charge and the mass of the particle. In particular, in the case of a particle being accelerated, we can show the existence of wave operators and its completeness for very weak decaying conditions of potentials V with respect to $|V(x)| \leq C(1 + |x|)^{-\rho}$ for some $\rho > 0$.

This talk is based on joint work with Tadayoshi Adachi ([1]).

References

[1] Adachi, T., Kawamoto, M.: Quantum scattering in a periodically pulsed magnetic field, (preprint) 2014.

January 22(Fri.) 14:30-15:20 **Taisuke Yoneyama** (Tokyo University of Science)

Title: An application of wave packet transform to scattering theory

Abstract

In this talk, we shall consider Schrödinger equation with time-dependent short-range potentials by introducing wave packet transform. Wave packet transform is defined by A. Córdoba and C. Fefferman. We will give the outline of the proof of the existence of the wave operators by wave packet transform and explain the difference between the characterization space in our study and that in H. Kitada–K. Yajima. In the proof, we use the representation with wave packet transform, which is developed by K. Kato, M. Kobayashi and S. Ito.