Weyl asymptotics with remainder for the transmission eigenvalues

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Abstract. We prove Weyl asymptotics $N(r) = cr^d + \mathcal{O}_\epsilon(r^{d-\kappa+\epsilon})$, $\forall\, 0 < \epsilon \ll 1$, for the counting function $N(r) = \sharp\{\lambda_j \in \mathbf{C} \setminus \{0\} : |\lambda_j| \le r^2\}$, $r > 1$, of the interior transmission eigenvalues (ITE), λ_j. Here d denotes the space dimension and $0 < \kappa \le 1$ is such that there are no (ITE) in the region $\{\lambda \in \mathbf{C} : |\mathrm{Im}\,\lambda| \ge C(|\mathrm{Re}\,\lambda| + 1)^{1-\frac{\kappa}{2}}\}$ for some $C > 0$. Note that a complex number $\lambda \in \mathbf{C}$, $\lambda \neq 0$, is said to be a transmission eigenvalue if the following problem has a non-trivial solution:

$$\begin{cases} (\nabla c_1(x)\nabla + \lambda n_1(x))\, u_1 = 0 & \text{in} \quad \Omega, \\ (\nabla c_2(x)\nabla + \lambda n_2(x))\, u_2 = 0 & \text{in} \quad \Omega, \\ u_1 = u_2, \ \ c_1\partial_\nu u_1 = c_2\partial_\nu u_2 & \text{on} \quad \Gamma, \end{cases}$$

where $\Omega \subset \mathbf{R}^d$, $d \ge 2$, is a bounded, connected domain with a C^∞ smooth boundary $\Gamma = \partial\Omega$, ν denotes the exterior Euclidean unit normal to Γ, and $c_j, n_j \in C^\infty(\overline{\Omega})$, $j = 1, 2$ are strictly positive real-valued functions.

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