Existence and non-existence of ground states in the translation invariant Nelson model

The Nelson model is a model from quantum field theory used to describe a spinless massive particle in interacting is a scalar quantum field. If there is no external potential, the model may be written as a direct integral of operators on fockspace of the form

$$H(\xi) = \Omega(\xi - d\Gamma(k)) + d\Gamma(\omega) + g\phi(v).$$

We give criteria for existence and non-existence of ground states for $H(\xi)$. In particular if one uses the physical choices of v, ω and Ω in dimension 3, then one can conclude that $H(\xi)$ has no ground state for any choice of ξ.