Semi-classical analysis of the interior Dirichlet-to-Neumann map

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Abstract. Let (X, g) be a compact Riemannian manifold with a non-empty smooth boundary ∂X and let Δ_X denote the negative Laplace-Beltrami operator on (X, g). Given a function $f \in H^{m+1}(\partial X)$, $m \geq 0$, let u solve the equation

$$\begin{cases} (\Delta_X + \lambda^2)\, u = 0 & \text{in} \quad X, \\ u = f & \text{on} \quad \partial X, \end{cases}$$

where $\lambda \in \mathbf{C}$, $1 \ll |\operatorname{Im} \lambda| \ll \operatorname{Re} \lambda$. Then the Dirichlet-to-Neumann (DN) map

$$N(\lambda) : H^{m+1}(\partial X) \to H^m(\partial X)$$

is defined by

$$N(\lambda) f := \partial_\nu u|_{\partial X},$$

where ν is the unit normal to ∂X. I will discuss the semi-classical structure of the operator $N(\lambda)$ with respect to the semi-classical parameter $h = |\lambda|^{-1}$. In particular, $N(\lambda)$ turns out to be an $h - \Psi DO$ in a good class as long as $|\operatorname{Im} \lambda| \geq h^{-1/2-\varepsilon}$, $0 < \varepsilon \ll 1$. I will also discuss some applications to the localization on the complex plane of the transmission egenvalues.

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